Starter Questions

Prove that $\dfrac{1}{\cos\theta+1}-\dfrac{1}{\cos\theta-1}\equiv\dfrac{2}{\sin^2\theta}$

Given that $\sin\theta=\sqrt{2}-1$, work out the values of the integers a and b such that $\cos^2\theta=a+b\sqrt{2}$

$\frac{1}{\cos\theta+1}-\frac{1}{\cos\theta-1}$		
$\equiv \frac{(\cos\theta-1)-(\cos\theta+1)}{(\cos\theta+1)(\cos\theta-1)}$	**M1**	Common denominator
$\equiv \frac{-2}{\cos^2\theta-1}$	**A1**	Correct numerator
$\equiv \frac{2}{1-\cos^2\theta} \equiv \frac{2}{\sin^2\theta}$	**M1**	Use of $\sin^2\theta+\cos^2\theta=1$
	A1	

$\cos^2\theta = 1-\sin^2\theta$	**M**	Use of $\cos^2\theta+\sin^2\theta=1$
$= 1-(\sqrt{2}-1)^2$	**M1**	Substitution of $\sin\theta$
$= 1-(3-2\sqrt{2})$		
$= -2+2\sqrt{2}$		
So $a=-2$, $b=2$	**A1**	

J Vectors

J3

Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- prove that two vectors are parallel to each other by showing that one is a multiple of the other
- recall the conditions for collinearity
- understand that a vector diagram can be used to find resultants. This could, for example, be in the context of force.

Note: unless specifically stated in a question, students can choose **any** appropriate method to solve vector problems.

6.1 Vector Definitions & Properties

Scalar – a quantity with magnitude (size) only

e.g. Speed → 10 m/s
Distance → 150m

Vector – a quantity with magnitude (size) and direction

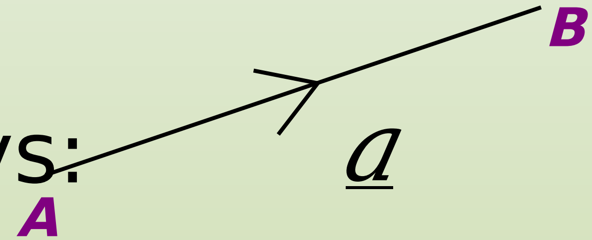
e.g. Velocity → 10 m/s on a bearing of 060^0
Displacement → 150m west

6.1 Vector Definitions & Properties

Notation

Consider the vector between points A and B

It can be written in two ways:



End points

Single letter   (must be underlined to indicate it is a vector. Shown in **bold** print in texts).

6.1 Vector Definitions & Properties

Magnitude

This is the size (or length) of a vector.
Magnitude uses the sign.

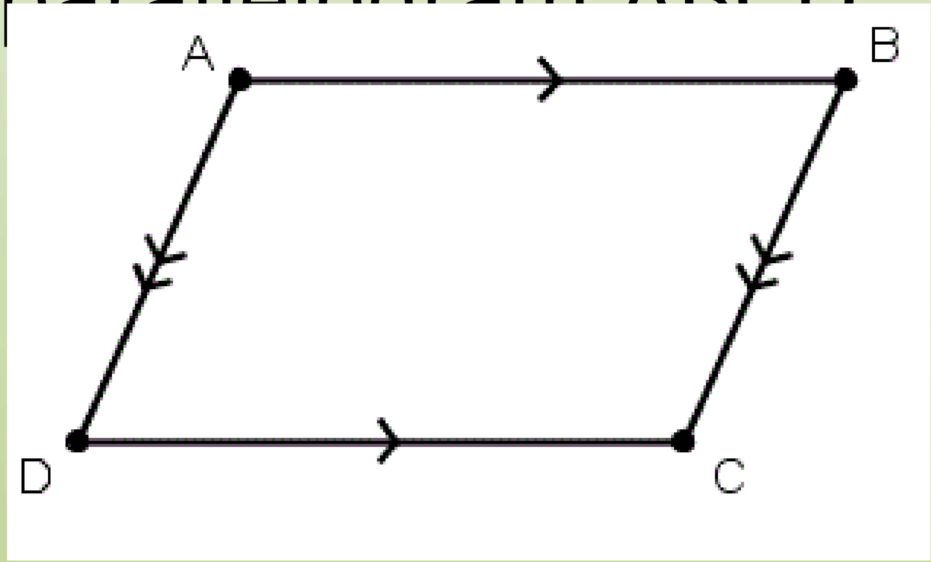
e.g. magnitude of

magnitude of

6.1 Vector Definitions & Properties

Equal Vectors

Equal vectors have the same magnitude and direction. Note that they don't have to have the same origin, e.g. consider the parallelogram ABCD.



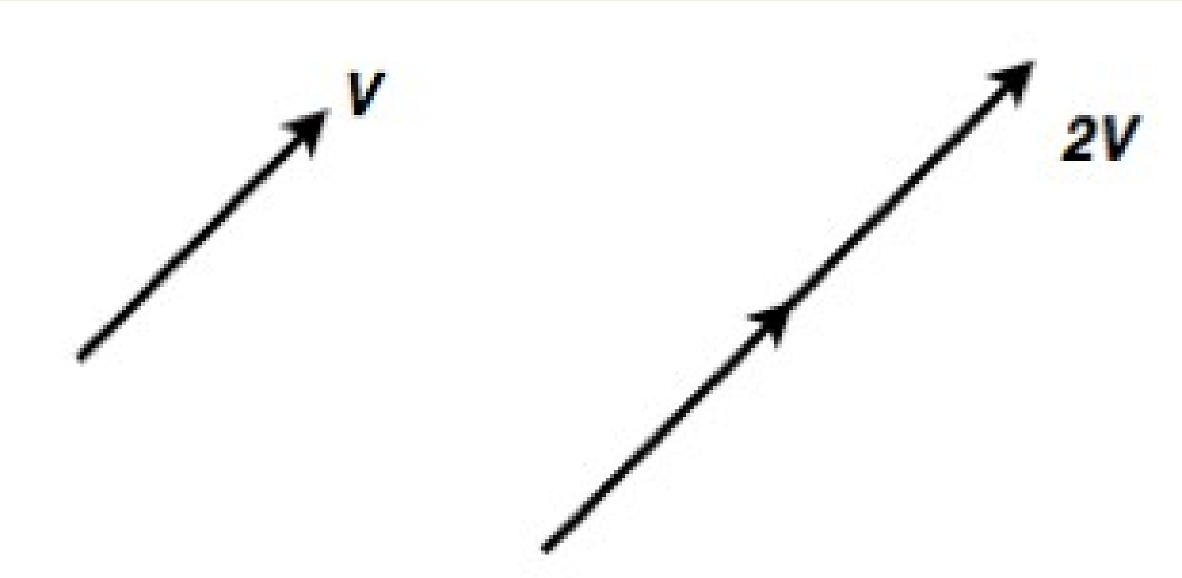
=

=

6.1 Vector Definitions & Properties

Scalar Multiplication

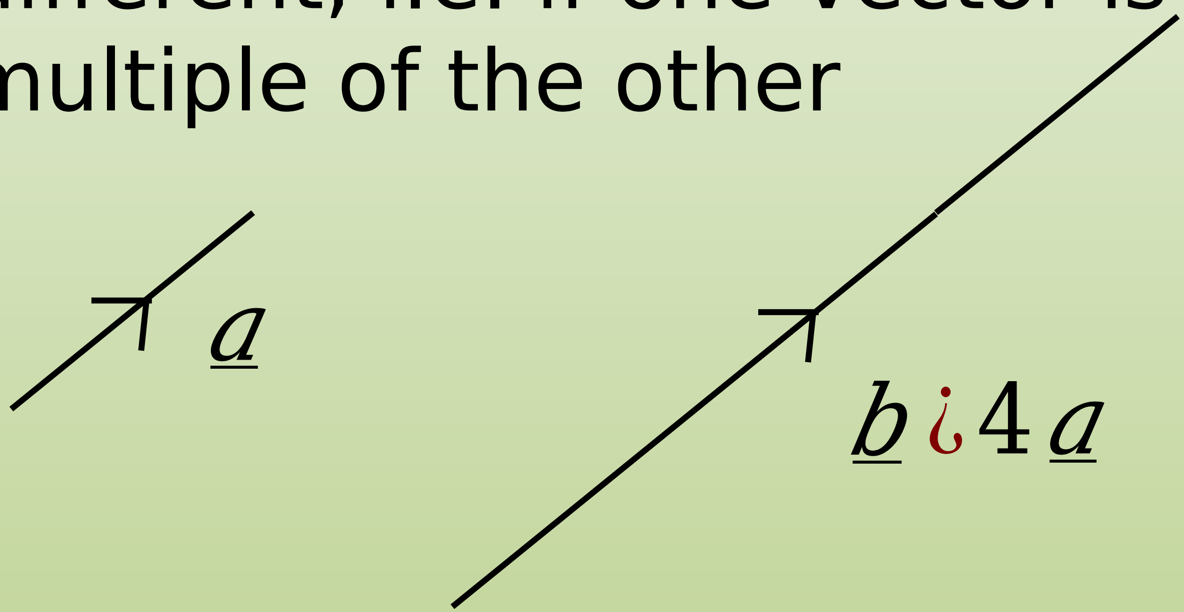
If a vector is multiplied by a number (scalar), then the magnitude is changed but not the direction.



6.1 Vector Definitions & Properties

Parallel Vectors

Vectors are parallel if they have the same direction. Their magnitudes can be different, i.e. if one vector is a scalar multiple of the other



is parallel to as is a scalar multiple of

Same direction

6.1 Vector Definitions & Properties

Parallel Vectors

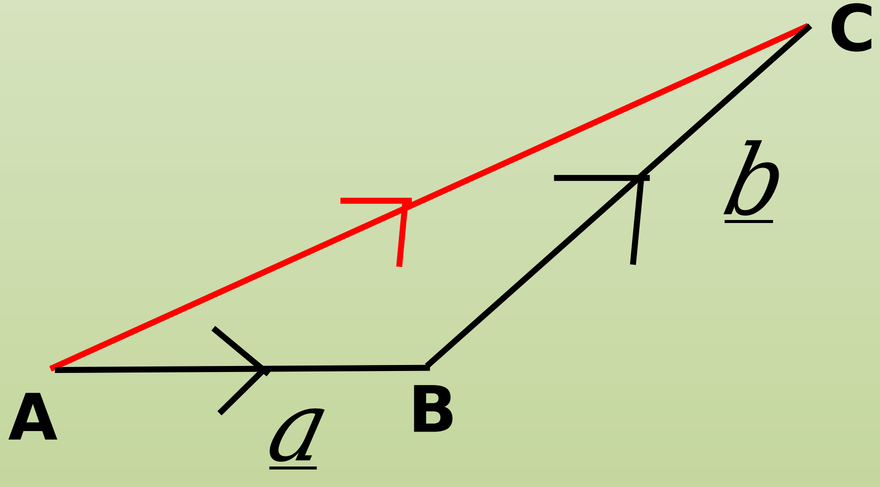
So if for some constant k, then is parallel to

6.1 Vector Definitions & Properties

Addition of Vectors

The vector sum of two or more vectors is known as their resultant. Drawing the vectors one after the other assists with finding the resultant.

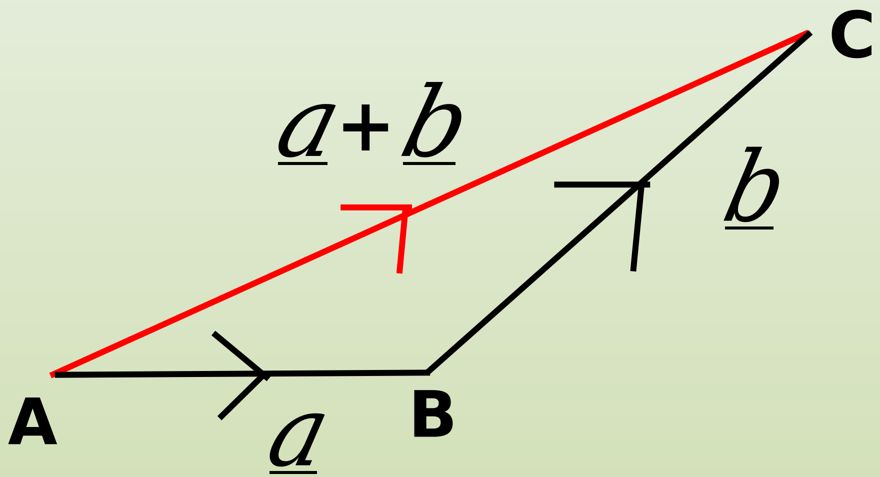
e.g.



is the resultant of adding and

6.1 Vector Definitions & Properties

Addition of Vectors



is the resultant of adding and

= +

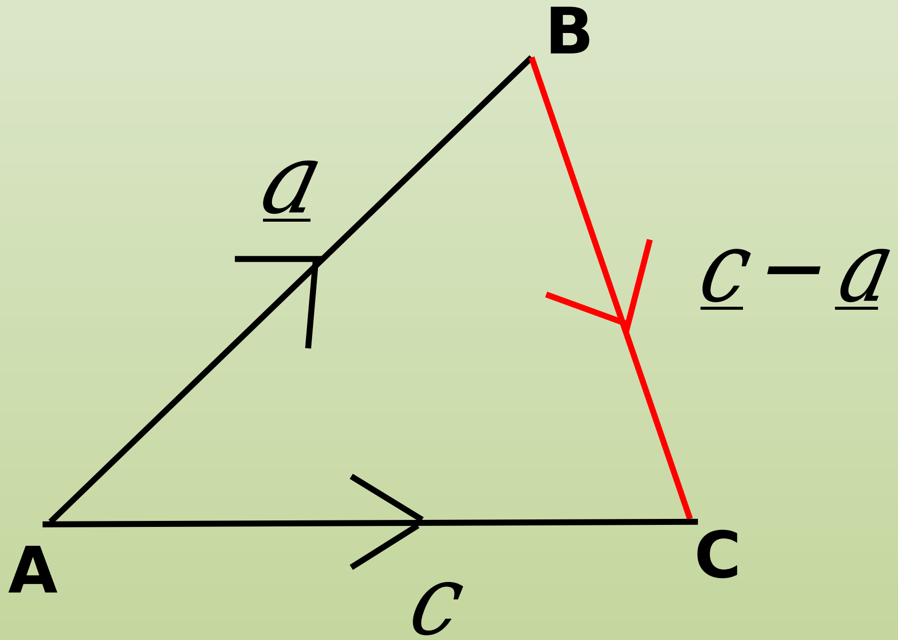
=

6.1 Vector Definitions & Properties

Subtraction of Vectors

Subtracting a vector is the same as adding its negative

e.g.



= +

= - +

= -

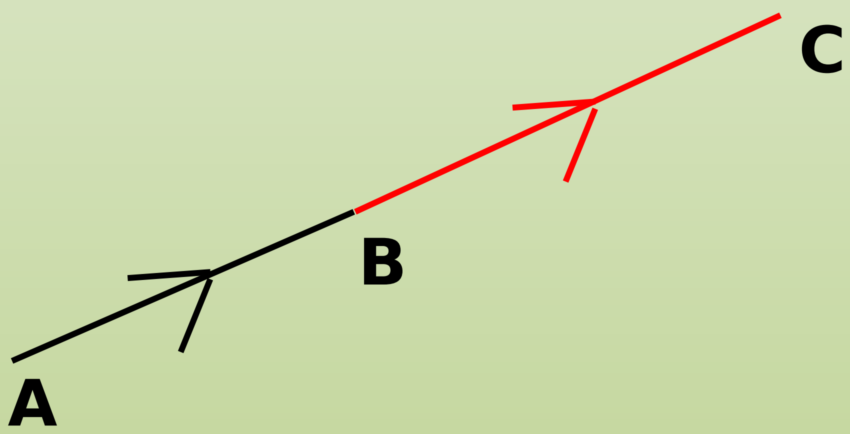
=

6.1 Vector Definitions & Properties

Collinearity

Two or more points are collinear (lie on the same straight line segment) if it can be shown that the vectors between them are parallel and that one point is common

e.g.



6.1 Vector Definitions & Properties

Unit Vector

A vector with a magnitude (length/size) of 1.

Unit vectors are denoted by .

A unit vector in the direction of is found by:

$$\boxed{=}$$

6.1 Vector Definitions & Properties

Zero Vector

A vector with no magnitude or direction.
Denoted by $\underline{0}$

e.g.

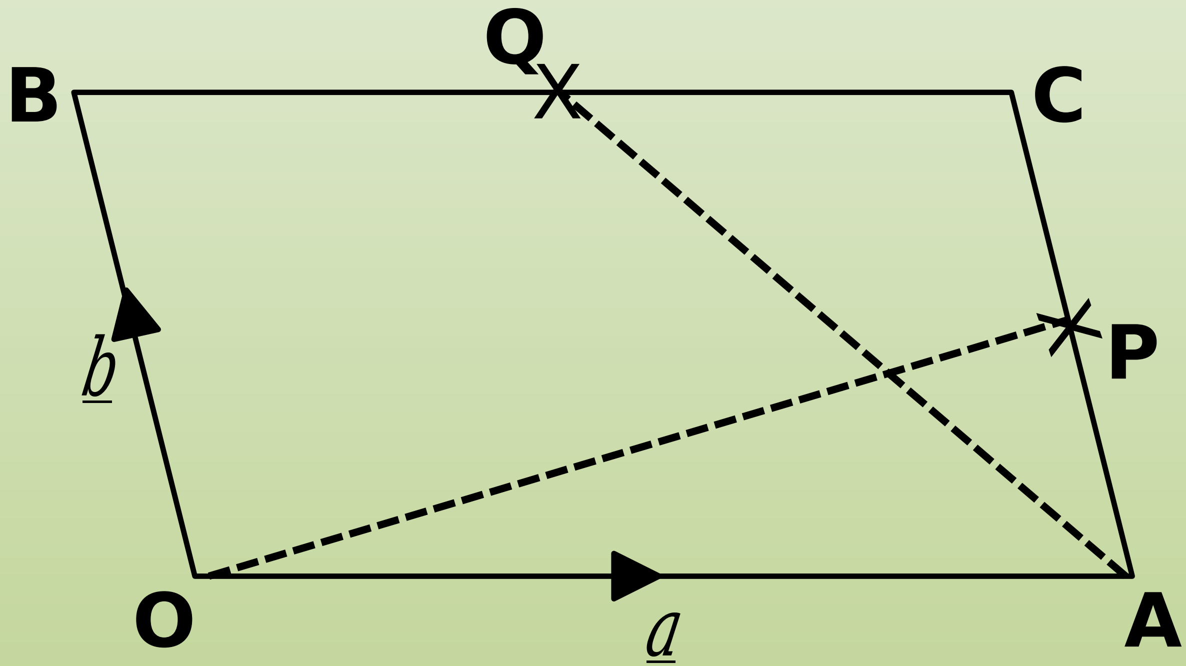
6.1 Vector Definitions & Properties

Example 1

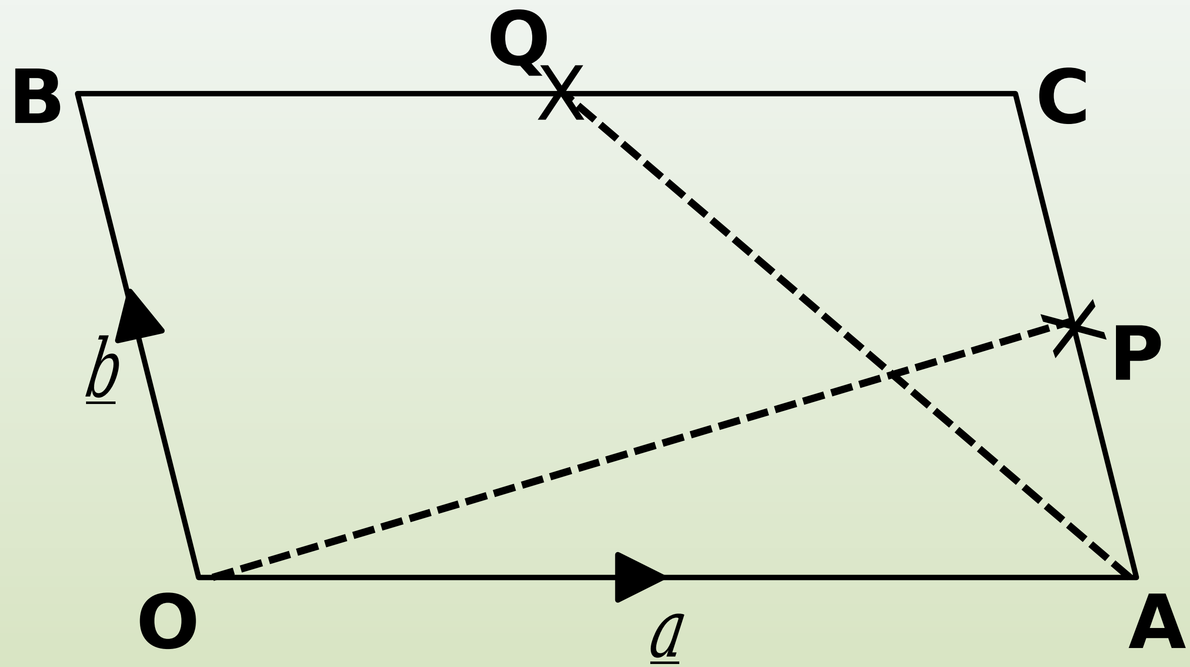
Consider the parallelogram OABC where P and Q are the midpoints of AC and BC respectively.

is vector

is vector



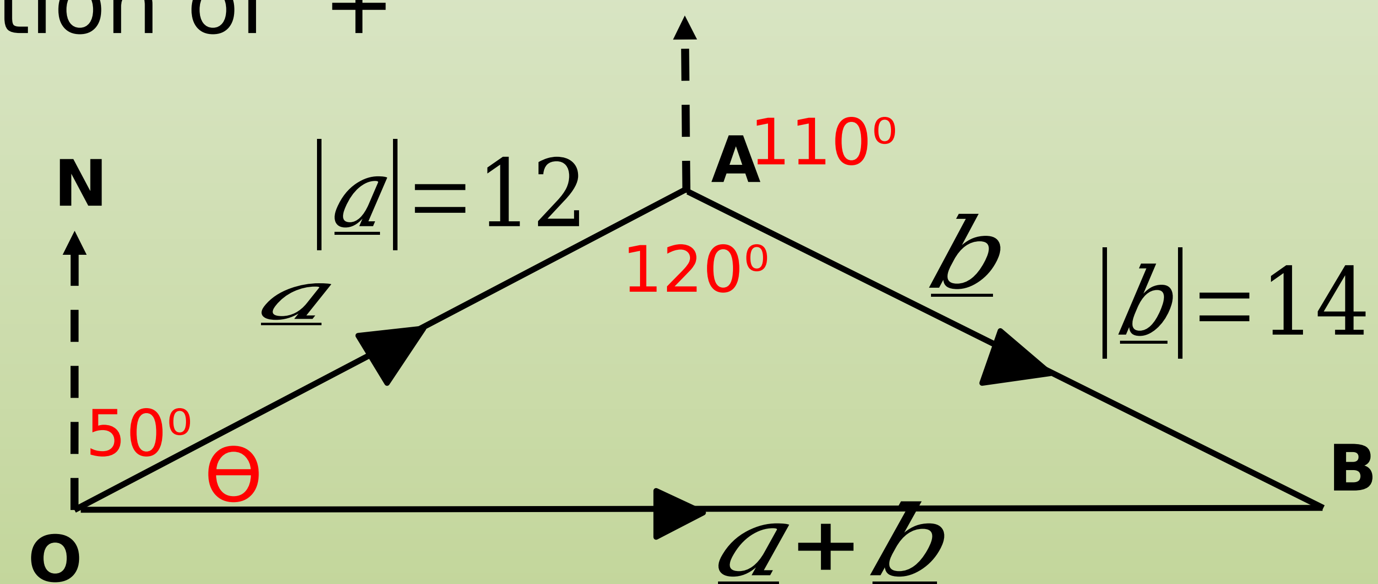
Write the following vectors in terms of and/or :



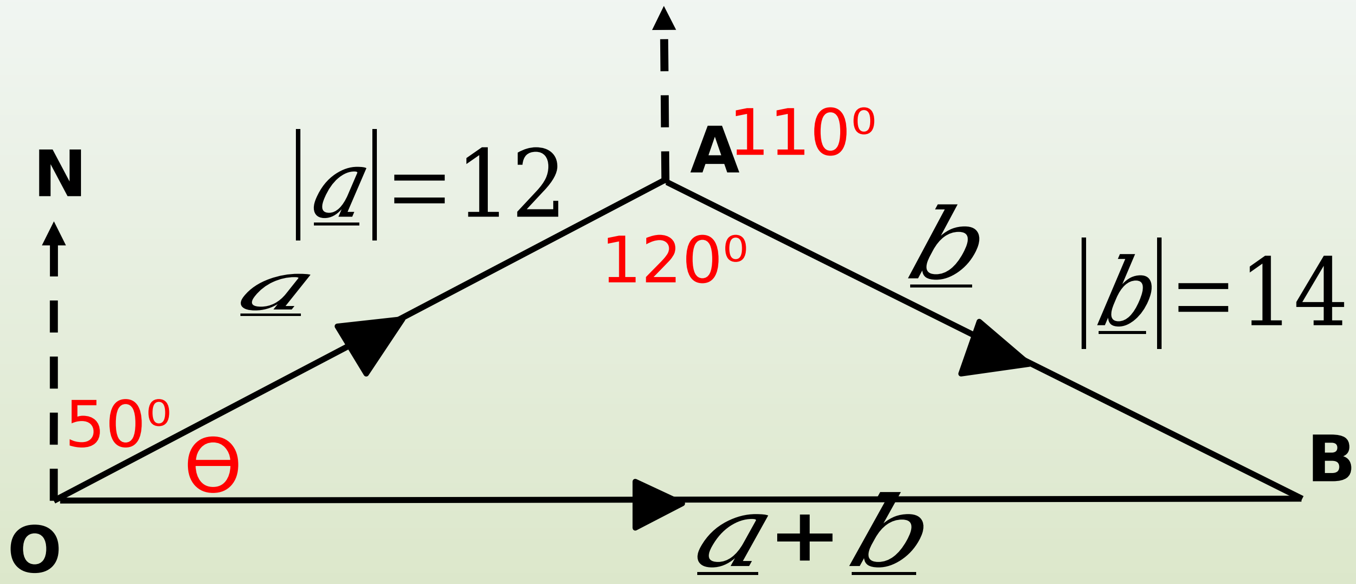
6.1 Vector Definitions & Properties

Example 2

Given that has a magnitude of 12 on a bearing of 050^0 and has magnitude 14 on a bearing of 110^0, find the magnitude and direction of +



...find the magnitude and direction of $\underline{a} + \underline{b}$



Using the cosine rule:

$OB^2 = OA^2 + AB^2 - 2(OA)(AB) \cos A$

$OB^2 = 12^2 + 14^2 - 2(12)(14) \cos 120$

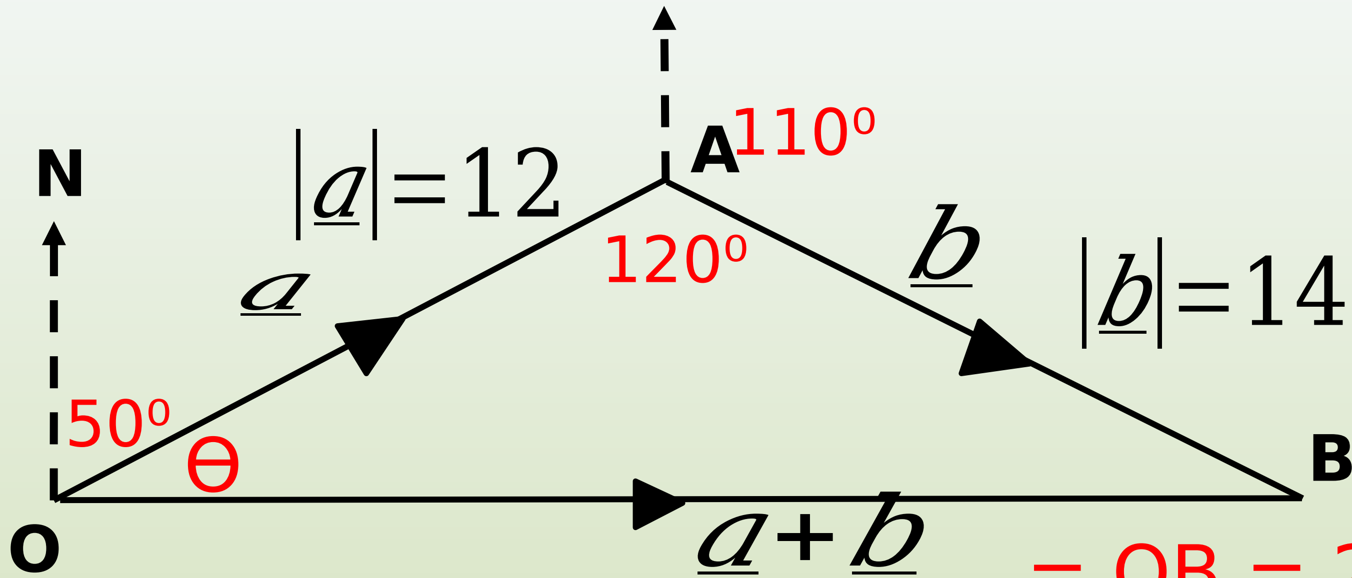
$OB^2 = 144 + 196 + 168$

$OB^2 = 508$

$OB = \sqrt{508} = 22.5$ (3sf)

$= OB = 22.5$

...find the magnitude and direction of +



= OB = 22.5

Using the sine rule:

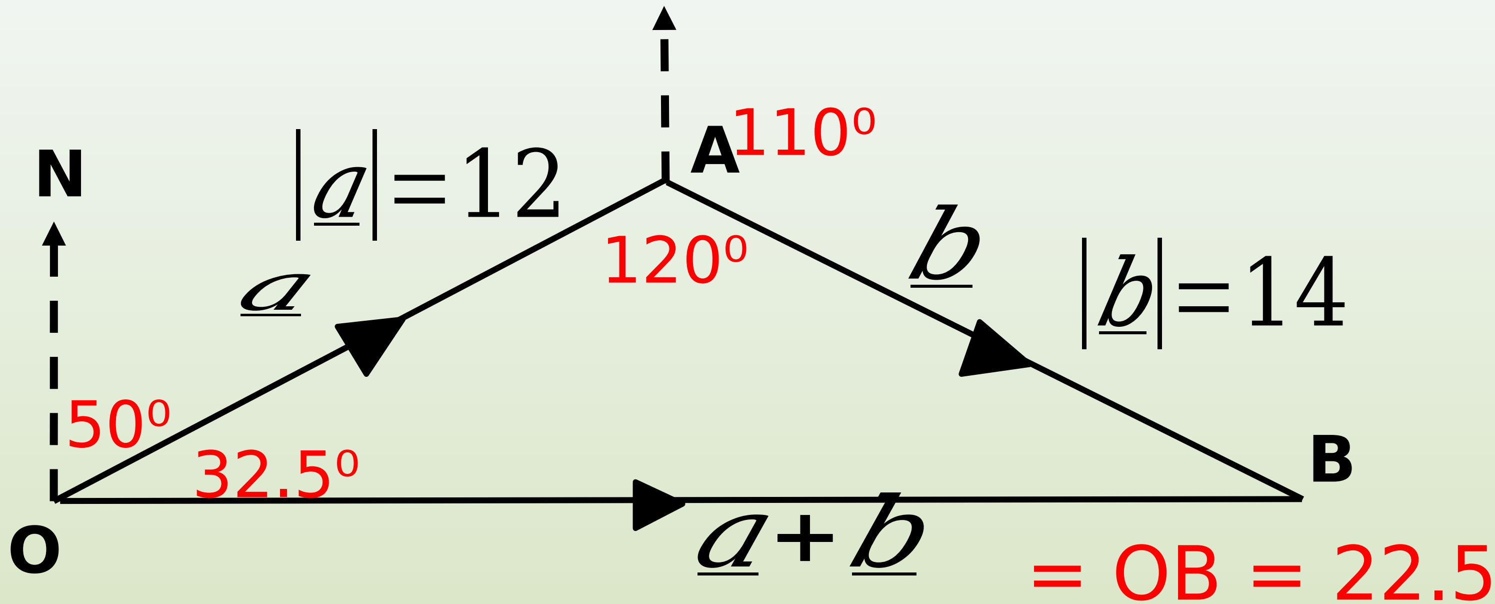
= 32.5 (3sf)

Bearing ():

= 050⁰ + 032.5⁰

= 083⁰

...find the magnitude and direction of +



$$\therefore magnitude\, of\ \underline{a} + \underline{b}\ is\ 22.5\ on\ a\ bearing\ of\ 083°$$

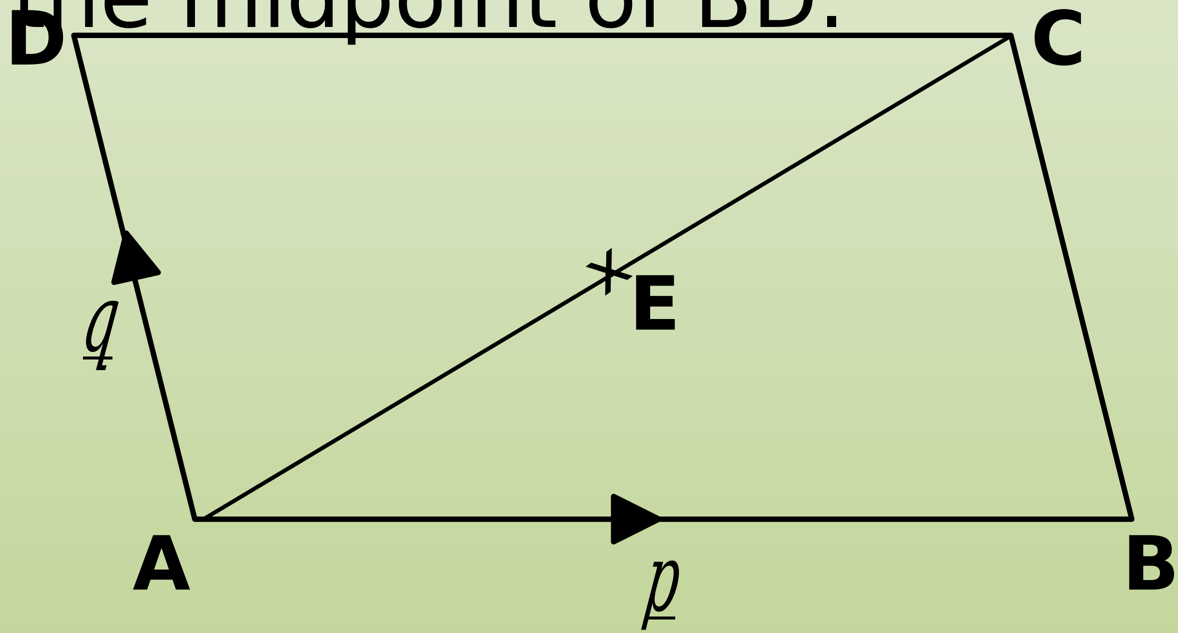
6.1 Vector Definitions & Properties

Example 3

ABCD is a parallelogram. E is the midpoint of AC.

Vector AB = **p** and vector AD = **q**.

Prove that E is the midpoint of BD.



= **p** + **q**

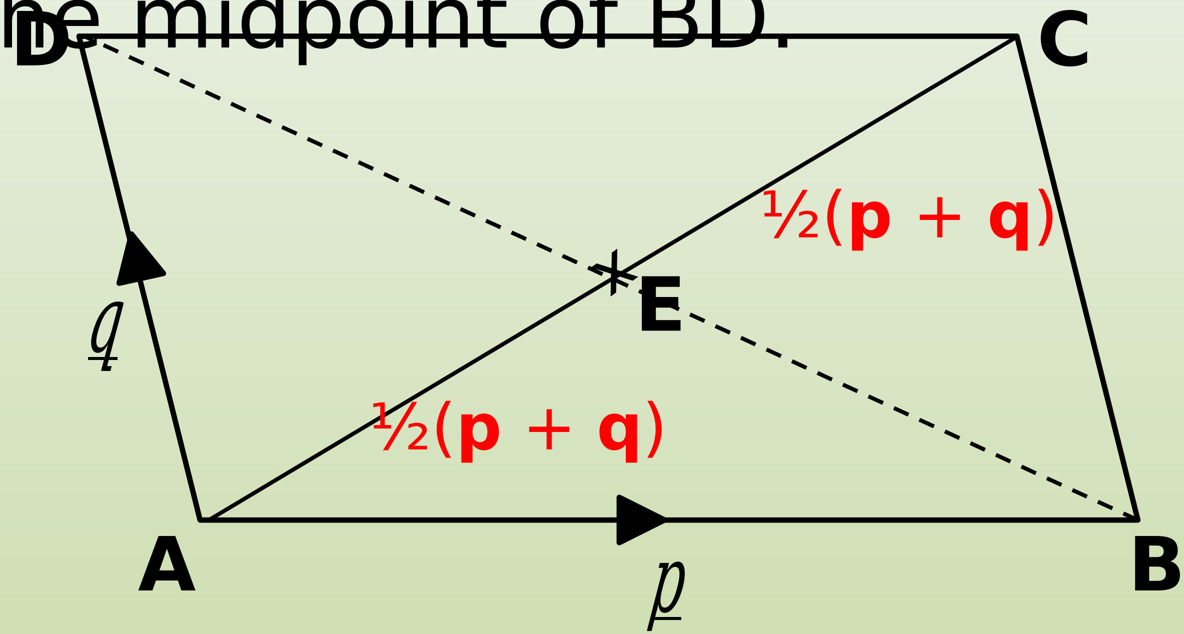
So = ½(**p** + **q**)

Example 3

ABCD is a parallelogram. E is the midpoint of AC.

Vector AB = **p** and vector AD = **q**.

Prove that E is the midpoint of BD.



$=\ +$

$= -\mathbf{p} + \tfrac{1}{2}(\mathbf{p} + \mathbf{q})$

$= \tfrac{1}{2}(\mathbf{q} - \mathbf{p})$

$= \mathbf{q} - \mathbf{p}$

So $= \tfrac{1}{2}$

Hence E is the midpoint of BD.

Example 4

The diagram shows parallelogram ABCD. E lies on DC, and DE:EC = 1:3. AE and BC, when extended, meet at F.

= +

= $\mathbf{q} + \frac{1}{4}\mathbf{p}$

AB = **p** and AD = **q**

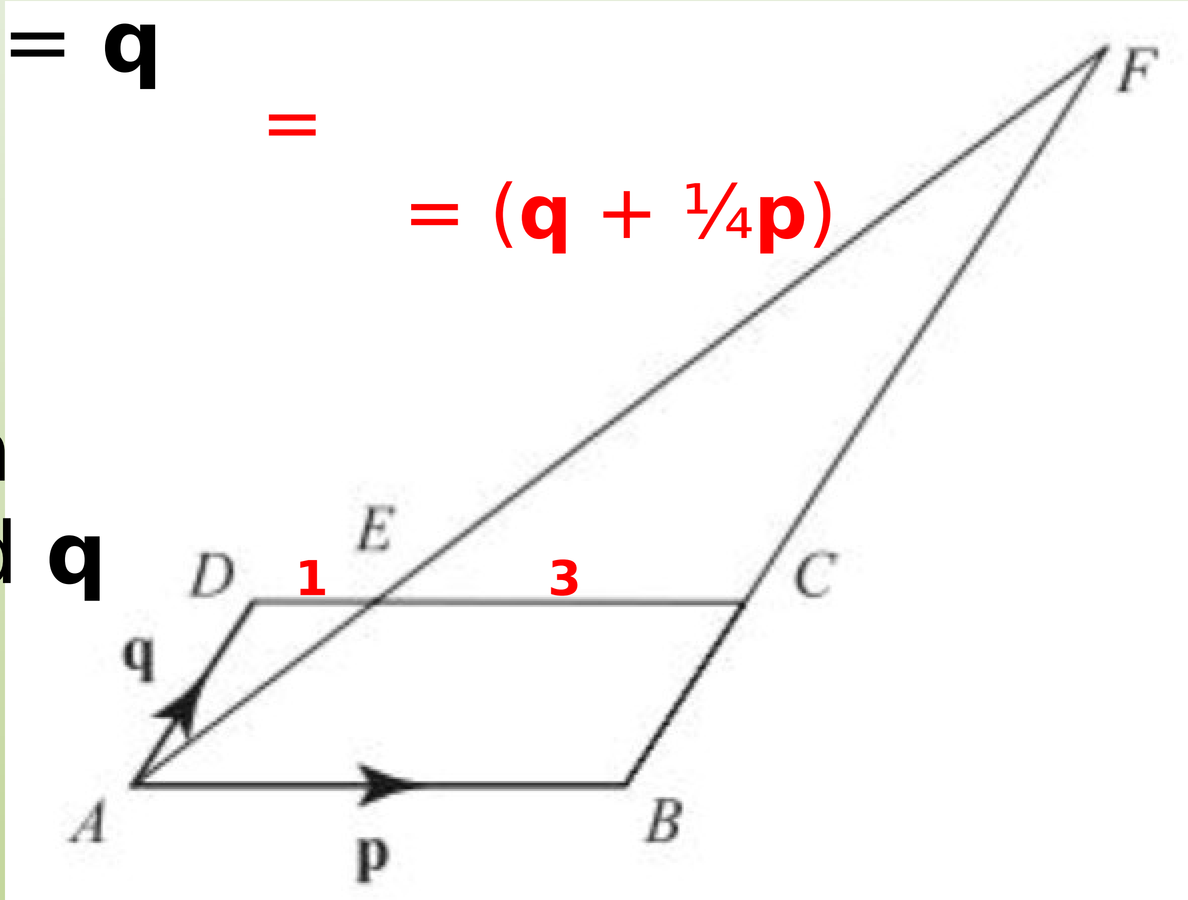
=

AF =

= $(\mathbf{q} + \frac{1}{4}\mathbf{p})$

BF =

a) Express AF in terms of , **p** and **q**



Example 4

The diagram shows parallelogram ABCD. E lies on DC, and DE:EC = 1:3. AE and BC, when extended, meet at F.

AB = **p** and AD = **q**

AF =

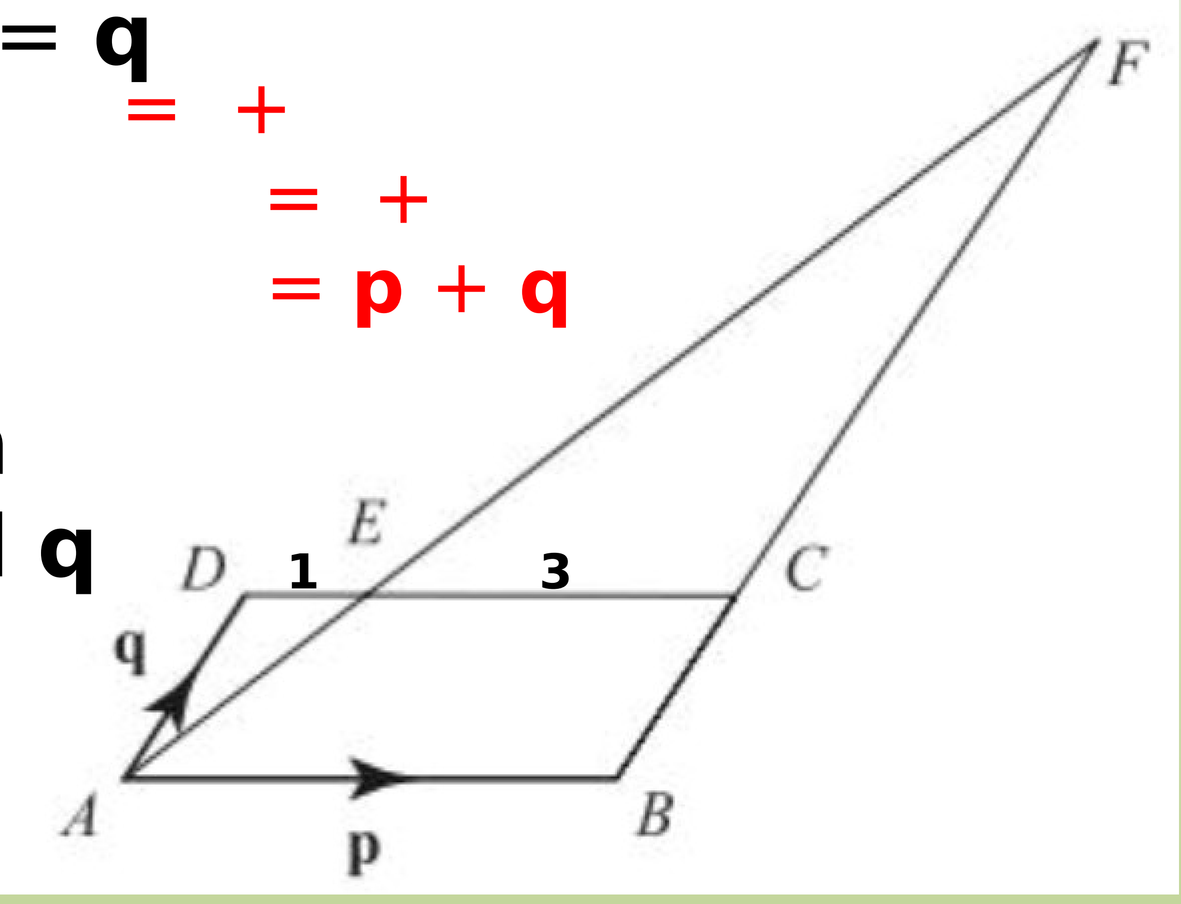
BF =

= +

= +

= **p** + **q**

b) Express AF in terms of , **p** and **q**



Example 4

The diagram shows parallelogram ABCD. E lies on DC, and DE:EC = 1:3. AE and BC, when extended, meet at F.

AB = **p** and AD = **q**

AF =

BF =

= (**q** + ¼**p**)

= **p** + **q**

(**q** + ¼**p**) = **p** + **q**

(¼ – 1)**p** = **q**

c) Hence find the values of

Since p and q are not parallel, ¼ – 1 = 0 and = 0.

This gives =4 and =4